(1)

differential equation

sepration of variable

problem 1 $(x+y)^2 \frac{dy}{dx} = a^2$

Soln:— we put $x+y = z$

$\Rightarrow 1 + \frac{dy}{dx} = \frac{dz}{dx}$

$\therefore \frac{dy}{dx} = \frac{dz}{dx} - 1$

Now from quetien

$z^2\left(\frac{dz}{dx} - 1\right) = a^2$

or $\frac{dz}{dx} - 1 = \frac{a^2}{z^2}$

$\Rightarrow \frac{dz}{dx} = \frac{z^2 + a^2}{z^2}$

$\Rightarrow dx = \frac{z^2}{z^2 + a^2} dz$

variable are separated so integrating both side

$\int dx = \int \frac{z^2 + a^2 - a^2}{z^2 + a^2} dz$

$\Rightarrow x + K = \int \left(1 - \frac{a^2}{z^2 + a^2}\right) dz$

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$$x+K = \int dt - a^2 \int \frac{dt}{t^2+a^2} dt$$

(2)

$$\Rightarrow x+K = t - a^2 \frac{1}{a} \tan^{-1}\frac{t}{a}$$

$$\Rightarrow x+K = x+y - a \tan^{-1}\frac{x+y}{ax}$$

$$[\because t = x+y]$$

$$\Rightarrow a \tan^{-1}\frac{x+y}{a} = y-K$$

$$\Rightarrow \tan^{-1}\frac{x+y}{a} = \frac{y-K}{a}$$

$$\therefore x+y = a \tan\left(\frac{y-K}{a}\right)$$

problem (2) Solve $\cos(x+y) = \frac{dx}{dy}$

(3) Soln: - put $x+y = t$

Diff. w.r.t y we get

$\frac{dx}{dy} + 1 = \frac{dt}{dy}$

$\Rightarrow \frac{dx}{dy} = \frac{dt}{dy} - 1$

So from quation

$\cos t = \frac{dt}{dy} - 1$

$\Rightarrow 1 + \cos t = \frac{dt}{dy}$

$\Rightarrow dy = \frac{dt}{1+\cos t}$

variables are separated

So integration both side

$\int dy = \int \frac{dt}{1+\cos t}$

$\Rightarrow y = \int \frac{dt}{2\cos^2 \frac{t}{2}} = \frac{1}{2}\int \sec^2 \frac{t}{2} dt$

$\Rightarrow y = \frac{1}{2} \cdot 2 \cdot \tan \frac{t}{2} + K$

or $y = \tan \frac{x+y}{2} = K$

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problem (3)

(4) solve $x\,dx + y\,dy = \dfrac{a^2(x\,dy - y\,dx)}{x^2+y^2}$

soln:- we put $x = r\cos\theta$

and $y = r\sin\theta$

$\Rightarrow \dfrac{y}{x} = \dfrac{r\sin\theta}{r\cos\theta} = \tan\theta$

and $x^2+y^2 = r^2\cos^2\theta + r^2\sin^2\theta$

$= r^2(\cos^2\theta + \sin^2\theta)$

$\Rightarrow x^2+y^2 = r^2$

~~$\Rightarrow x^2+y$~~

$\Rightarrow 2x\,dx + 2y\,dy = 2r\,dr$

$\Rightarrow x\,dx + y\,dy = r\,dr$

and from $\dfrac{y}{x} = \tan\theta$

on diff we get

$\dfrac{x\,dy - y\,dx}{x^2} = \sec^2\theta\,d\theta$

using these relation in question we get

$r\,dr = \dfrac{a^2x^2\sec^2\theta\,d\theta}{r^2}$

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(5) $\Rightarrow r\,dr = \dfrac{a^2 r^2 \cos 2\theta \sec 2\theta \, d\theta}{r^2}$

$\Rightarrow r\,dr = a^2 d\theta$

Variable are separabled

$\int r\,dr = a^2 \int d\theta$

$\dfrac{r^2}{2} = a^2\theta + K$

$\Rightarrow \dfrac{1}{2}(x^2+y^2) = a^2 \tan^{-1}\dfrac{y}{x} + K$

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